MATHEMATICAL MODEL OF HYDRODYNAMICS FOR FILLING CLOSED VOLUMES WITH ACCOUNT OF AIR INJECTION

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A mathematical model of hydrodynamics is proposed for filling closed volumes with allowance for air capture by a liquid jet. Adequacy of the model is established by comparison between the calculated and experimental data obtained by means of physical modeling.

It has been noted on physical models that when filling a confined volume by a liquid from above, its hydrodynamics are essentially exposed to air captured by a jet [1, 2]. In this case depending on the amount of air the flow pattern may vary qualitatively.

The data of physical modeling [3] testify the fact that under the real conditions of filling steel-pouring ladles and moulds from above, the amount of air captured by a liquid metal jet is rather significant and its volume may reach the volume of the entering metal. However, in the available mathematical models of hydrodynamic processes, occurring when filling ladles and moulds [4-6], this fact is not taken into account. This situation is, probably, associated with purely technical difficulties in mathematical simulation of a two-phase gas–liquid medium [7].

In the present work we suggest a rather simple mathematical model of hydrodynamics for filling closed volumes with regard to air capture by a metal jet. The adequacy of the proposed model is established by comparison between the calculated and experimental data obtained through physical modeling [8].

The mathematical model for hydrodynamic processes involving a gaseous phase has been proposed earlier in [9] for the case of flow-through of a ladle by an inert gas. In this model two fields of velocities were taken into account: liquid velocities $\mathbf{V}_1$ and averaged velocities of the gaseous phase (gas bubbles $\mathbf{V}_2$). The two-phase interaction force was determinated by their relative velocity $\mathbf{V}_{21} = \mathbf{V}_2 - \mathbf{V}_1$:

$$\mathbf{F}_{21} = N\pi r_n^2 C_\mu \frac{\rho_0 V_{21}^2}{2} \frac{\mathbf{V}_{21}}{V_{21}},$$

here ρ_0 is the liquid density; C_μ is the hydrodynamic resistance coefficient, in this case the velocity $\mathbf{V}_{21}$ is vertically directed upward and its magnitude is expressed by the semiempirical formula:

$$V_{21} = \sqrt{\frac{\sigma}{r_n(\rho_0 - \rho)} + g r_n\left(1 - \frac{\rho}{\rho_0}\right)}(1 - \alpha)^{\tilde{\nu}},$$

where $\alpha = {}^4/_3 \pi r_n^3 N$ is the gas content coefficient; σ is the surface tension coefficient of the liquid; $\rho = \rho_0(1 - \alpha)$ is the gas–liquid mixture density; $g = 9.8$ m/sec^2.

The presence of two empirical parameters C_μ and $\tilde{\nu}$ (which, in addition, depend on the flow pattern) significantly degrades the usefulness of the two-velocity model in practical computations because there are not any reliable numerical data for C_μ and $\tilde{\nu}$. In this case, following the authors of [9], we may use numerical experimentations and fit parameters C_μ and $\tilde{\nu}$ in such a way to finally obtain correct values of the experimentally verified quantities, for example, of velocity fields for the physical model. However, some other parameters, for instance, the turbulence parameters (on which C_μ and $\tilde{\nu}$ are also dependent)

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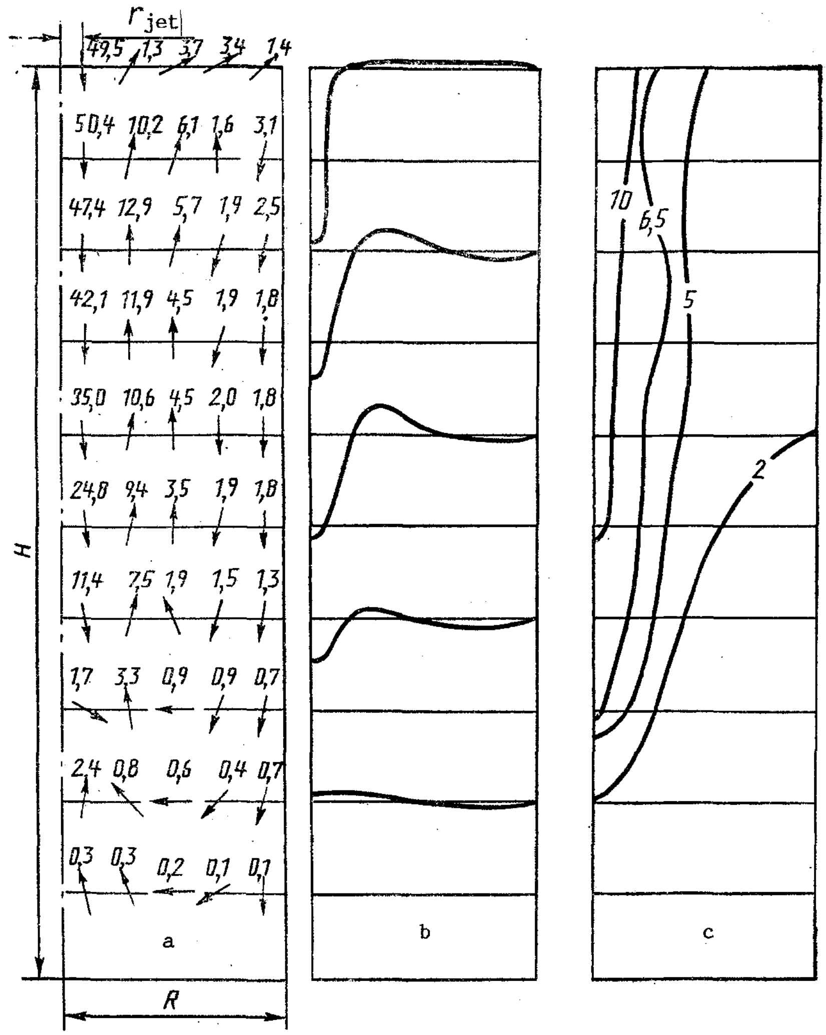

Fig. 1. Calculation results on filling the vessel with account of air injection by the jet at V_0 = 1 m/sec and $\alpha_0 = 0.1$; a) field of velocities with indication of their values, cm/sec; b) curves of vertical velocities, V_z; c) lines of air isoconcentration, %.

exert an effect on the velocity fields, therefore it is practically impossible to assess the contribution of one or the other parameter to the final pattern. Moreover, because of the problem nonlinearity complicated by the nonlinear dependence of C_μ and $\tilde{\nu}$ on the motion parameters, it is rather difficult to extend these results to other regions that can not be immediately verified experimentally.

That is why in the present work we formulate the mathematical model which eliminates the need to the maximum possible extent to use the difficult-to-determine parameters and which takes into account only the main physical factors affecting the flow pattern of the two-phase gas–liquid medium in the conditions of our problem.

Contrary to the authors of [9], we do not describe transfer of the gaseous and liquid phases separately with two different velocity fields for each of phases, but in the combined one-velocity approach via the supposition on the continuity of the single gas–liquid medium, being density-stratified by the viscous incompressible liquid. In this case there is no longer a need to make any suppositions concerning the shape and sizes of the gas bubbles.

The motion equations for such medium take on the form [10]

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\nabla)\,\mathbf{V} = -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\,\Delta \mathbf{V} + \mathbf{g}; \tag{1}$$

$$\nabla \mathbf{V} = 0; \tag{2}$$

$$\frac{\partial \rho}{\partial t} + (\mathbf{V}\nabla)\rho = 0. \tag{3}$$

Here **V** is the velocity vector of the medium which is determined as the ratio of the pulse density π of the medium to mass density ρ: $\mathbf{V} = \pi/\rho$. Vector **V** in the general case coincides neither with liquid velocity $\mathbf{V}_1$, nor with gas velocity $\mathbf{V}_2$, but it is the mean between them (coinciding with them in the particular case, $\mathbf{V}_1 = \mathbf{V}_2$), and characterizes the gas–liquid medium flow as a unit.

Let us next assume that the main factor, defining the flow pattern of the gas–liquid medium when filling the closed volume, is the buoyancy force $\mathbf{f} = -\alpha\mathbf{g}$ arising due to the nonuniform density caused by the presence of gaseous inclusions. This assumption allows us to write Eq. (1) in the Boussinesq approximation:

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\nabla)\mathbf{V} = -\nabla\tilde{p} + \nu\Delta\mathbf{V} + (1-\alpha)\mathbf{g}, \tag{4}$$

where $\tilde{p} = p/\rho_0$ and $\nu = \mu/\rho_0$. Substituting the relation $\rho = \rho_0(1 - \alpha)$ into Eq. (3), we may replace it by the equivalent equation for the gas content coefficient α:

$$\frac{\partial \alpha}{\partial t} + (\mathbf{V}\nabla)\alpha = 0. \tag{5}$$

When deriving Eq. (5) α is assumed to depend explicitly only on the spatial point and time. From the physical viewpoint this means that cavitation phenomena, compressibility of bubbles, and other processes are neglected here.

Equations (2), (4), and (5) are the complete system of equations for evaluating the all necessary parameters in the gas–liquid medium flow, i.e., **V**, $\tilde{p}$, and α. They must be added by the boundary conditions corresponding to our problem.

Now suppose that the filled volume has a cylindrical form and the jet enters into it along the symmetry axis. This allows us to arrange the boundary conditions in the symmetrical manner. Therefore the half of the cylinder axial section (Fig. 1a) can be chosen here as the computational domain.

The boundary conditions for the velocity on the symmetry axis and the vessel walls are chosen by the conditions of nonleakage and free slipping

$$V_{\perp} = 0; \ \mathbf{n}\cdot\nabla V_{\parallel} = 0, \tag{6}$$

where **n** is the unit normal vector to the surface, while on the free surface they are chosen by the conditions of the incoming (leaving) flow,

$$V_{\perp} = V_s, \ \mathbf{n}\cdot\nabla V_{\parallel} = 0, \tag{7}$$

here V_s takes on the value in the region of the incoming jet, i.e., of the jet velocity V_0, whereas on the remaining part of the surface – of the rise velocity of liquid mirror V_ζ. The boundary conditions for the pressure are obtained by projecting Eq. (4) onto the normal to the surface. Upon the gas content coefficient α on the symmetry axis and vessel walls the nonleakage condition is superimposed

$$\mathbf{n}\cdot\nabla\alpha = 0, \tag{8}$$

and on the surface – the free leakage condition

$$\alpha = a, \tag{9}$$

with $a = \alpha_0$ in the jet and $a = 0$ on the liquid mirror.

In order to take into account the turbulent character of the flow, we introduce the effective coefficients of viscosity and diffusion of the gaseous phase bubbles including turbulent and approximation components [10]. The stated problem is numerically solved in natural variables by the splitting method with respect to physical factors. In our case this method may be realized in the following manner. Let at the time instant $t_n = n\tau$ (where τ is the time pitch and n is the number of pitches) the velocity fields **V** and gas content α be known. Then at time instant $t_{n+1} = (n + 1)\tau$ these functions as well as pressure $\tilde{p}$ may be determined by the three-stage splitting scheme

$$\text{I:}\ \frac{\tilde{\mathbf{V}} - \mathbf{V}^n}{\tau} = -(\mathbf{V}^n\nabla)\mathbf{V}^n + \nu\Delta\mathbf{V}^n + (1-\alpha^n)\mathbf{g}; \tag{10}$$

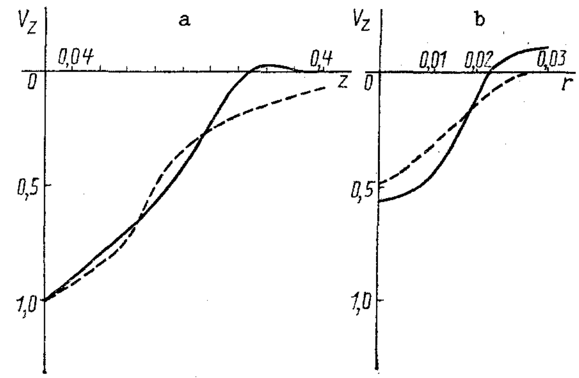

Fig. 2. Calculated and experimental data for vertical velocities V_z, m/sec; z, r, m.

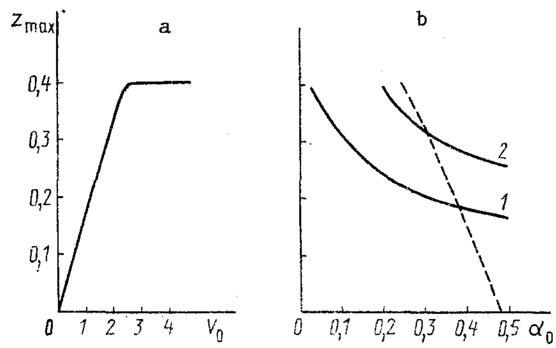

Fig. 3. Dependence of the jet penetration depth on initial jet velocity V_0 (a) and air concentration α_0 (b). z_{max}, m; V_0, m/sec.

$$\frac{\tilde{\alpha} - \alpha^n}{\tau} = -(\mathbf{V}^n \nabla)\,\alpha^n; \tag{11}$$

$$\mathrm{II}:\ \Delta\tilde{p} = (\nabla\tilde{\mathbf{V}})/\tau; \tag{12}$$

$$\mathrm{III}:\ \frac{\mathbf{V}^{n+1} - \tilde{\mathbf{V}}}{\tau} = -\nabla\tilde{p}; \tag{13}$$

$$\frac{\alpha^{n+1} - \alpha^n}{\tau} = -(\mathbf{V}^n \nabla)\,\tilde{\alpha}. \tag{14}$$

This scheme is the combination of Belotserkovskii's splitting scheme with respect to physical factors for the hydrodynamic equations [10] and of Nikitenko's scaled difference scheme [11] for the equation of convective transfer of the gas concentration and these have some common advantages.

The difference analog of Eqs. (10)-(14) written in cylindrical coordinates, is plotted on a staggered grid in the standard fashion [10, 11].

The computation program is realized in the Pascal language. Specific calculations were performed on an IBM PC AT/386 computer.

To establish the adequacy of the proposed model, a set of computations was carried out for different air-filled conditions of the cylindrical vessel with radius R = 0.1 m and height H = 0.4 m, because for this case there are some sufficiently reliable experimental data [8].

Figure 1a exhibits the predicted velocity field obtained for the jet of radius $r_{jet} = 0.01$ m at the incoming velocity under the surface $V_0 = 1$ m/sec and injected-air fraction $\alpha_0 = 0.1$. It is seen from Fig. 1a that our model describes qualitatively correctly the experimentally observed flow pattern radically distinguished from that which takes place in the absence of air injection. So, the jet penetrates not to the bottom of the vessel but only to definite depth r_{max} and it is retarded, in addition, by the buoyancy force acting on air bubbles in the jet, which float up and develop upward-directed flow in the neighborhood of the jet. In what follows, this flow reaches the surface and moves along it toward the vessel walls, simultaneously becomes free of gaseous inclusions, freely leaving the liquid volume through the surface. The flow, however, continues to travel downwards along the vessel walls until joins up with the incoming flow near the bottom. Figure 1b shows the curves of vertical velocities obtained by way of calculations at different levels.

Thus, in our case the vortex rotation direction of the gas-liquid medium is opposite to the direction of its rotation without regard to the jet-injected air.

However, a pattern like this is not always observed; it is detected only under certain conditions for the geometrical and hydrodynamic parameters of the motion. It is clear, for example, that at $\alpha_0 = 0$ this pattern cannot arise no matter what the other parameters of the system may be.

The calculated lines of the equal gaseous phase concentration in the liquid volume are given in Fig. 1c (the numbers express the values of the gas content coefficient in percent). As we might expect, the largest concentration of the gaseous phase is observed in the jet penetration region and it sharply decreases at the walls and at the bottom of the vessel.

The quantitative comparison between the computational and experimental data for vertical velocity component V_z was carried out in two directions: the vertical direction (along the symmetry axis (Fig. 2a)) and the radial direction (at a depth of $z = 0.16$ m (Fig. 2b)). In Fig. 2 the calculated data are depicted by the solid curves and the data of experiments from [8] – by the dashed curves. The comparison of these curves shows that their distinction does not exceed $0.15V_0$. This characterizes the adequacy degree of the proposed mathematical model.

The greatest distinction between the calculated and experimental curves is manifested in their "tail" (at large z). This is associated with the fact that in work [8] no account has been taken of the dependence of maximum jet penetration depth z_{max} on its initial velocity V_0, which was revealed during the numerical experiments and confirmed by observations. This dependence is given in Fig. 3a for the above-described geometrical parameters of the system and $\alpha_0 = 0.3$. At the incoming velocity $V_0 = 2.5$ m/sec of the jet it reaches the vessel bottom, but the vortex rotation direction in this case remains unchanged up to the velocity $V_0 = 5$ m/sec when the kinetic energy of the jet takes on the critical value, and the jet, splitting against the vessel bottom and moving along it, still possesses sufficient energy for reaching the vessel walls along which it then rises up to the surface. Here there occurs the qualitative change in the flow pattern: the vortex rotation motion is reversed and corresponds to the direction of the liquid vortex rotation in the absence of air injection. With these very reasons we associate the distinction of the dependence of jet penetration depth z_{max} on initial gas content coefficient α_0 calculated by our model and by the formulae from [8] (Fig. 3b). If for all V_0 from [8] there follows the common linear dependence of z_{max} on α_0 (dashed curve in Fig. 3b), then in accordance with our model for different velocities V_0 we have different nonlinear dependences of $z_{max}(\alpha_0)$ (solid curves in Fig. 3b given for $V = 1$ m/sec (1) and 1.9 (2)).

CONCLUSIONS

1. The proposed mathematical model over wide ranges of the change in parameters describes adequately the hydrodynamics of filling a vessel from above with air injection by the liquid jet taken into account and may be applied, for example, to the numerical investigation of actual conditions of filling ladles and moulds from above.

2. Air, injected by the liquid jet, exerts an essential effect on the liquid hydrodynamics and consideration must be given to it during mathematical simulation.

NOTATION

α_0, α, μ, ν, coefficients of gas content in a jet, gas content of a medium, dynamic and kinematic viscosity; **f**, buoyancy force; g, free fall acceleration vector; H, vessel height; N, a number of gas bubbles of r_n radius per unit volume; p, pressure; r_{jet}, r_n, R, radii of the jet, gas bubbles, and the vessel; t, time; **V**, $\mathbf{V}^n$, $\mathbf{V}_1$, $\mathbf{V}_2$, $\mathbf{V}_{21}$, velocity vectors of the gas–liquid medium on the n-th time pitch, liquid, gaseous phase, and of the relative interphase; V_0, V_ζ, values of velocities of the jet, and the metal mirror; z_{max}, maximum jet penetration depth into the vessel; z, r, vertical and horizontal coordinates; $\tilde{\nu}$, parameter depending on flow conditions; π, medium pulse density.

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